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Inversion of moveout velocities for horizontal transverse isotropy

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ABSTRACT

Transverse isotropy with a horizontal axis of symmetry (HTI) is the simplest azimuthally anisotropic model used to describe fractured reservoirs that contain parallel vertical cracks. Here, I present an exact equation for normal-moveout (NMO) velocities from horizontal reflectors in HTI media and apply it to invert moveout data for the anisotropic parameters. The azimuthal dependence of P–wave NMO velocity, that can be obtained from 3–D surveys, provides enough information to determine the principal direction of the anisotropy (crack orientation) and the P–wave vertical velocity, as well as an effective anisotropic parameter equivalent to Thomsen's coefficient δ.

The parameter of fracture systems of most interest in exploration is the crack density that is usually estimated through the traveltimes or reflection amplitudes of the split shear waves at vertical incidence. The formalism developed here makes it possible to obtain the crack density using the NMO velocities of P and shear waves from horizontal reflectors. Furthermore, the crack density can be estimated just from the P–wave NMO velocity in the special case of the vanishing parameter ϵ corresponding to thin cracks and negligible equant porosity. Also, P–wave moveout alone is sufficient to constrain the crack density if either dipping events are available or the velocity in the symmetry direction is known. For a more stable inversion, P–wave NMO velocities can be combined with the azimuthal dependence of amplitude variation with offset (AVO) response and the results of shear-wave polarization analysis.

Wave propagation in the symmetry plane of HTI media that contains the symmetry axis is described by the same equations as for vertical transverse isotropy (VTI media). Although the parameters of the "equivalent" VTI medium are extremely uncommon (e.g., $\epsilon < 0$), time-related processing in this symmetry plane is governed by the same two effective parameters as for vertical transverse isotropy. The anisotropic coefficients recovered from moveout data can also be used for processing in off-symmetry planes that requires a more elaborate treatment.

The approach used here to derive the NMO equation for horizontal transverse isotropy can be generalized for horizontal and dipping reflectors in more complicated azimuthally anisotropic models including off-symmetry planes in orthorhombic media.

INTRODUCTION

In horizontally-layered, isotropic media, normal-moveout (NMO) velocity of reflected waves is equal to the root-mean-square of the velocities in each layer. Conventional velocity analysis takes advantage of this simple relation by obtaining interval velocities from the NMO velocity via Dix (1955) formula. If the medium is anisotropic, normal-moveout velocity even in a single layer is no longer equal to the vertical velocity. The difference between the vertical and moveout velocity in anisotropic formations, such as shales, causes errors in time-to-depth conversion (Banik, 1984). On the other hand, inversion of moveout velocities can provide estimates of the anisotropic coefficients that can be used in seismic processing, amplitude variation with offset (AVO) analysis, and lithology discrimination.

Analytic expressions for NMO velocities from horizontal reflectors are well known for transversely isotropic media with a vertical symmetry axis, or vertical transverse isotropy (VTI) (e.g., Lyakhovitsky and Nevsky 1971; Hake et al., 1984; Thomsen 1986). Using Thomsen's (1986) notation, the NMO velocities of the P–, SV–, and SH–waves [1] in a single VTI layer can be represented as

$$V_{\rm nmo}\,[P\text{—wave}] = V_{Pvert}\sqrt{1+2\delta^{(\rm V)}}\,, \tag{1}$$

$$V_{\rm nmo}\,[SV\text{—wave}] = V_{Svert}\sqrt{1+2\sigma^{(\rm V)}}\,, \tag{2}$$

$$V_{\rm nmo}\,[SH\text{—wave}] = V_{Svert}\sqrt{1+2\gamma^{(\rm V)}}\,, \tag{3}$$

with

$$\sigma^{(\rm V)} \equiv \left(\frac{V_{Pvert}}{V_{Svert}}\right)^2 (\epsilon^{(\rm V)} - \delta^{(\rm V)})\,, \tag{4}$$

where V_{Pvert} and V_{Svert} are the vertical velocities of the P- and S–waves respectively, $\epsilon^{(\rm V)}$, $\delta^{(\rm V)}$, and $\gamma^{(\rm V)}$ are the Thomsen's anisotropy parameters for vertical transverse isotropy, and $\sigma^{(\rm V)}$ is the effective parameter introduced by Tsvankin and Thomsen (1994) to describe SV–wave propagation. Equations (1)–(3) are valid for VTI media with arbitrary strength of the anisotropy.

As discussed by Tsvankin and Thomsen (1995), the NMO velocities from horizontal reflectors in VTI media are not sufficient to recover the vertical velocities and anisotropic parameters, even if all three waves are recorded. However, if some additional information is available (such as the reflector depth or one of the vertical velocities), equations (1)–(3) make it possible to obtain the anisotropic coefficients.

Another practically important anisotropic model is transversely isotropic media with a horizontal symmetry axis, or horizontal transverse isotropy (HTI). The most common physical reason for HTI symmetry is a system of parallel vertical cracks

[1] I will omit the qualifiers in "quasi-P–wave" and "quasi-SV–wave."

(fractures) embedded in an isotropic matrix (Crampin, 1985; Thomsen, 1988). It should be emphasized that while modeling and processing of reflection data are more complicated for horizontal transverse isotropy than for VTI media, the azimuthal dependence of moveout velocities and amplitudes in HTI models provides additional information for the inversion procedure. Obviously, this inversion is impossible without relating the attributes of the reflected waves to the anisotropic parameters.

Thomsen (1988) presented the weak-anisotropy approximation for NMO velocities of $P-$ and $S-$waves from a horizontal reflector in the symmetry plane of HTI media that contains the symmetry axis. More general nonhyperbolic ("skewed") moveout equations for pure modes both in the symmetry and off-symmetry planes were given by Sena (1991); his results, however, are valid only for weak anisotropy and horizontal reflectors. A weak-anisotropy formalism similar to that by Sena (1991) was employed by Li and Crampin (1993) to study the moveout from horizontal reflectors in transversely isotropic and orthorhombic media.

Here, I present an exact equation for normal-moveout velocities of pure modes valid for any orientation of the survey line over an HTI layer. If the anisotropy is caused by vertical cracks, $P-$wave moveout data can be used to find the crack orientation and estimate the crack density – an important parameter in reservoir characterization. It is also shown that time processing of $P-$wave data in the plane that contains the symmetry axis is governed by the same two effective parameters that Alkhalifah and Tsvankin (1995) introduced for vertical transverse isotropy.

ANISOTROPY PARAMETERS FOR HTI MEDIA

Horizontal transverse isotropy can be characterized by the stiffness matrix c_{ij} or Thomsen's (1986) parameters in the rotated coordinate system with the x_3 axis pointing in the symmetry direction. The relation between the Thomsen parameters and the stiffness coefficients in this coordinate frame is the same as for vertical transverse isotropy:

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}}, \tag{5}$$

$$V_{S0} \equiv \sqrt{\frac{c_{55}}{\rho}}, \tag{6}$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}, \tag{7}$$

$$\delta \equiv \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, \tag{8}$$

$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}}, \tag{9}$$

where ρ is the density. The parameters V_{P0} and V_{S0}, which in VTI models correspond to the vertical P and S–wave velocities respectively, in our case represent the P and S–wave velocities in the symmetry (horizontal) direction.

Two vertical symmetry planes of HTI media, shown in Figure 1, will be referred to as the "isotropy plane" (the plane normal to the symmetry axis) and the "symmetry plane" (the plane that contains the symmetry axis). Note that the wave propagation in the isotropy plane can be described by the isotropic equations because the velocities of all three waves do not change with direction. As we will see later on, the velocities and polarizations in the symmetry plane can be found by analogy with VTI media.

The shear waves will be denoted as "$S^{\parallel}$" and "$S^{\perp}$," with the $S^{\parallel}$–wave polarized in the isotropy plane and the $S^{\perp}$–wave polarized in the plane formed by the symmetry axis and the ray. The form of the superscripts is explained by the fact that in HTI media due to parallel vertical cracks the polarization vector of $S^{\parallel}$ is parallel to the crack planes, while the wave $S^{\perp}$ at vertical incidence is polarized normal to the cracks. In the symmetry plane the $S^{\perp}$–wave represents an in-plane (SV) motion while the $S^{\parallel}$–wave is polarized in the direction orthogonal to the plane and may be called the SH–wave. Therefore, for this plane the $S^{\perp}$– and $S^{\parallel}$–waves can be denoted as the SV– and SH–waves, respectively. However, the polarizations of the shear waves recorded in any other plane do not conform to this simple rule. For instance, in the isotropy plane the particle motion of the wave that we refer to as $S^{\parallel}$ will be confined to the incidence plane, while the $S^{\perp}$–wave is polarized parallel to the symmetry axis and, therefore, orthogonally to the incidence plane. The wave $S^{\parallel}$ is often called the "fast" shear wave since at vertical incidence and in the isotropy plane it propagates faster than $S^{\perp}$.

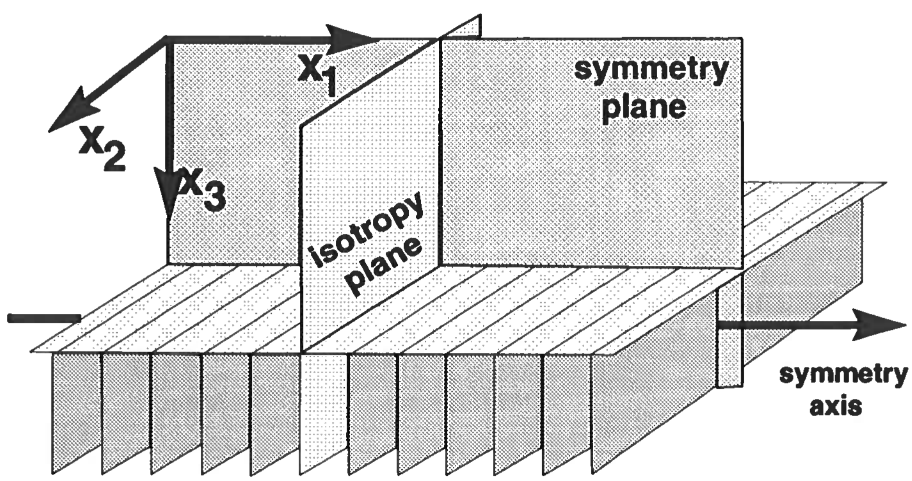

FIG. 1. Two vertical symmetry planes in HTI media. Wave propagation in the plane that contains the symmetry axis can be described by analogy with vertical transverse isotropy. In the plane normal to the symmetry axis ("isotropy plane"), velocity is independent of propagation angle.

The slowness surface of the wave $S^{\perp}$ can be obtained from the slowness surface of the SV-wave for vertical transverse isotropy by a 90-degree rotation; the same is true for the $S^{\|}$- and SH-waves. Hence, the velocities and traveltimes of the waves P and $S^{\perp}$ for horizontal transverse isotropy are determined by the same four coefficients as for $P-SV$-waves in VTI media (V_{P0}, V_{S0}, ϵ, and δ). Furthermore, Thomsen notation makes it possible to reduce the number of parameters that control P-wave kinematic signatures from four to three: P-wave velocities and traveltimes depend mostly just on V_{P0}, ϵ, and δ (Tsvankin and Thomsen, 1994; Tsvankin, 1995b). It should also be mentioned that the parameters introduced by equations (5)–(9) are convenient to use in TI media with any magnitude of velocity variations, not just for weak transverse isotropy (for a detailed discussion, see Tsvankin, 1995b).

EQUIVALENCE BETWEEN VERTICAL AND HORIZONTAL TRANSVERSE ISOTROPY

The results of moveout analysis for vertical transverse isotropy can be extended to the vertical plane that contains the symmetry axis in HTI media (we call it the "symmetry plane") by using the equivalence between vertical and horizontal transverse isotropy. By the "equivalent" VTI model I will mean the VTI medium that can be used to describe velocities, traveltimes, and polarizations of body waves in the symmetry plane of the original HTI model. To obtain the parameters of this equivalent model, it is sufficient to examine the Kelvin-Christoffel matrix G_{ik} that determines (along with the density) the velocities and polarization vectors of plane waves. Let us find G_{ik} for wave propagation in the $[x_1, x_3]$ plane of a transversely isotropic medium with the axis of symmetry pointing in the x_1 direction (Figure 2). We will denote the stiffness tensor for this model by $c_{ijkl}^{(\mathrm{V})}$ to distinguish it from the tensor c_{ijkl} that corresponds to the rotated coordinate system with the x_3 axis coinciding with the symmetry direction [see equations (5)–(9)]. The transformation of one stiffness tensor into the other can be accomplished by interchanging the indices 1 and 3. Using the matrix notation (Voigt recipe), we find the following transformation rule for the independent stiffness components:

$$c_{11}^{(\mathrm{V})} = c_{33}; \;\; c_{33}^{(\mathrm{V})} = c_{11}; \;\; c_{13}^{(\mathrm{V})} = c_{13}; \;\; c_{55}^{(\mathrm{V})} = c_{55}\,, \tag{10}$$

and

$$c_{44}^{(\mathrm{V})} = c_{66}; \;\; c_{66}^{(\mathrm{V})} = c_{44}\,. \tag{11}$$

The Kelvin-Christoffel matrix is given by

$$G_{ik} = c_{ijkl}^{(\mathrm{V})}\, m_j m_l\,,$$

where $\vec{m}$ is the slowness vector that we consider confined to the $[x_1, x_3]$ plane. The non-zero components of the Kelvin-Christoffel matrix are expressed through the elastic constants $c_{ij}^{(\mathrm{V})}$ as

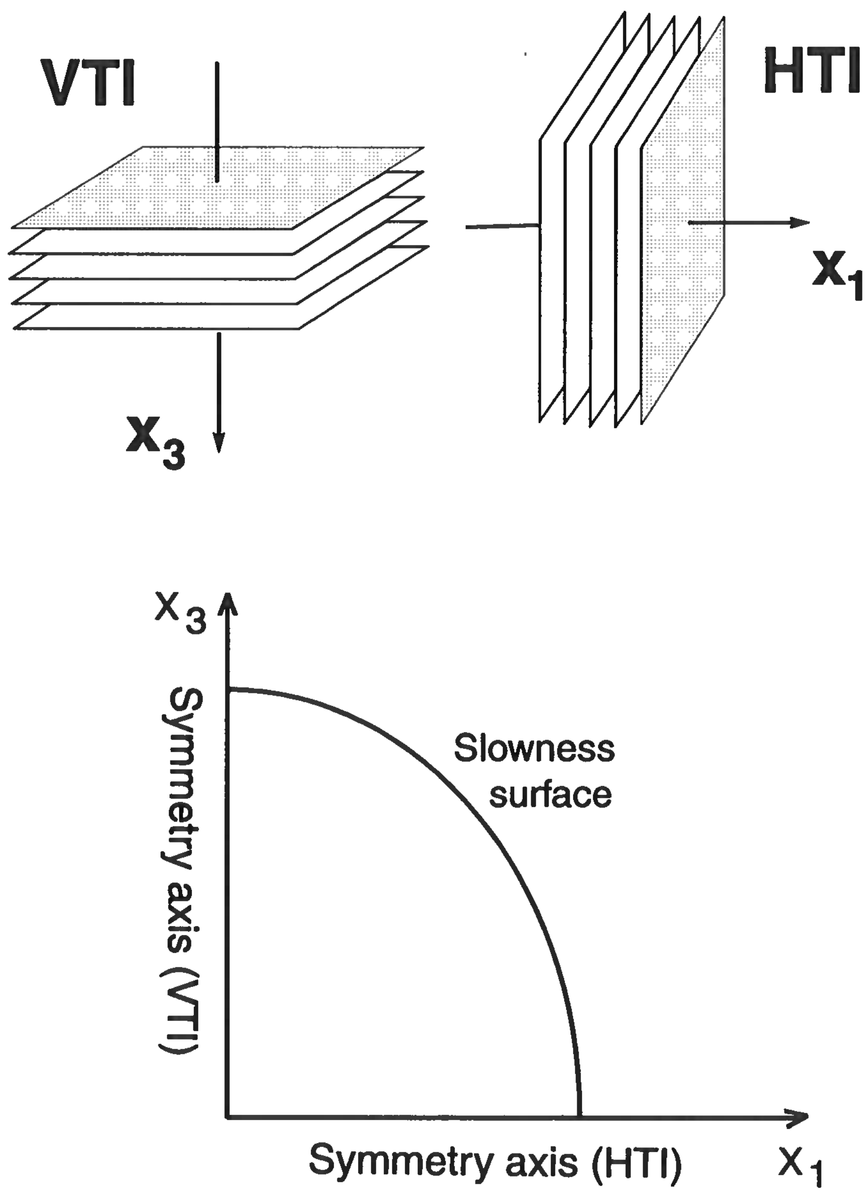

FIG. 2. Symmetry plane $[x_1, x_3]$ of a transversely isotropic medium with the symmetry axis pointing either in the x_3 (VTI) or in the x_1 (HTI) direction. The slowness surface in the symmetry plane remains the same if we replace the HTI medium with the equivalent VTI model.

$$G_{11} = c_{11}^{(\mathrm{V})} m_1^2 + c_{55}^{(\mathrm{V})} m_3^2 \,, \tag{12}$$

$$G_{33} = c_{33}^{(\mathrm{V})} m_3^3 + c_{55}^{(\mathrm{V})} m_1^2 \,, \tag{13}$$

$$G_{13} = (c_{13}^{(\mathrm{V})} + c_{55}^{(\mathrm{V})})\, m_1 m_3 \,, \tag{14}$$

$$G_{22} = c_{66}^{(\mathrm{V})} m_1^2 + c_{44}^{(\mathrm{V})} m_3^2 \,. \tag{15}$$

It is easy to verify that equations (12) – (15) are identical to the corresponding expressions for G_{ik} in the $[x_1, x_3]$ plane of a medium with the symmetry axis pointing in the x_3 direction. This means that wave propagation in the symmetry plane of HTI media that contains the symmetry axis can be studied using the equations for vertical transverse isotropy.

The kinematic properties and polarizations of $P - S^{\perp}$–waves are fully determined by the components G_{11}, G_{33}, and G_{13}, whereas the wave $S^{\|}$ is dependent only on G_{22}. Therefore, the velocities and traveltimes of the waves P and $S^{\perp}$ in the $[x_1, x_3]$ plane will be identical for the symmetry axis pointing either in the x_1 (HTI) or x_3 (VTI) direction, provided the stiffnesses $c_{11}^{(\mathrm{V})}$, $c_{33}^{(\mathrm{V})}$, $c_{55}^{(\mathrm{V})}$ and $c_{13}^{(\mathrm{V})}$ [equation (10)] are the same for both models. Likewise, the $S^{\|}$–wave velocity would be the same in the $[x_1, x_3]$ plane of VTI and HTI media if $c_{44}^{(\mathrm{V})}$ and $c_{66}^{(\mathrm{V})}$ [equation (11)] are fixed. Note that if the medium is horizontally transversely isotropic with a symmetry axis pointing in the x_1 direction, $c_{ij}^{(\mathrm{V})}$ from equations (10) and (11) do not specify the same VTI model (it is clear from the fact that $c_{44}^{(\mathrm{V})} \neq c_{55}^{(\mathrm{V})}$). However, since $P - S^{\perp}$– and $S^{\|}$–waves in the symmetry plane are decoupled, we can just use two different VTI models to describe $P - S^{\perp}$ and $S^{\|}$ propagation. One VTI model, designed for $P - S^{\perp}$ waves, will be characterized by $c_{11}^{(\mathrm{V})}$, $c_{33}^{(\mathrm{V})}$, $c_{55}^{(\mathrm{V})}$ and $c_{13}^{(\mathrm{V})}$, while the other ("$S^{\|}$") model will include $c_{44}^{(\mathrm{V})}$ and $c_{66}^{(\mathrm{V})}$.

Thus, wave propagation in the plane that contains the symmetry axis in HTI media can be described by the known VTI equations using the constants $c_{ij}^{(\mathrm{V})}$, or, alternatively, by the corresponding Thomsen parameters that we denote $\epsilon^{(\mathrm{V})}$, $\delta^{(\mathrm{V})}$, and $\gamma^{(\mathrm{V})}$:

$$\epsilon^{(\mathrm{V})} \equiv \frac{c_{11}^{(\mathrm{V})} - c_{33}^{(\mathrm{V})}}{2c_{33}^{(\mathrm{V})}} \,, \tag{16}$$

$$\delta^{(\mathrm{V})} \equiv \frac{(c_{13}^{(\mathrm{V})} + c_{55}^{(\mathrm{V})})^2 - (c_{33}^{(\mathrm{V})} - c_{55}^{(\mathrm{V})})^2}{2c_{33}^{(\mathrm{V})}(c_{33}^{(\mathrm{V})} - c_{55}^{(\mathrm{V})})} \,, \tag{17}$$

$$\gamma^{(V)} \equiv \frac{c_{66}^{(V)} - c_{44}^{(V)}}{2c_{44}^{(V)}} . \tag{18}$$

These coefficients of the equivalent VTI medium can be expressed through the original Thomsen parameters given by equations (7), (8), and (9) using equations (10) and (11):

$$\epsilon^{(V)} = -\frac{\epsilon}{1+2\epsilon} , \tag{19}$$

$$\delta^{(V)} = \frac{\delta - 2\epsilon\left(1+\frac{\epsilon}{f}\right)}{(1+2\epsilon)\left(1+\frac{2\epsilon}{f}\right)} , \tag{20}$$

$$\gamma^{(V)} = -\frac{\gamma}{1+2\gamma} , \tag{21}$$

where

$$f \equiv 1 - V_{S0}^2/V_{P0}^2 \tag{22}$$

is a useful parameter introduced by Tsvankin (1995b); V_{P0} and V_{S0} are the velocities in the symmetry (horizontal) direction.

The $S^{\perp}$–wave coefficient σ should be transformed according to

$$\sigma^{(V)} = \frac{\sigma}{1+\frac{2\epsilon}{f}} . \tag{23}$$

Similarly, the P–wave vertical velocity used in VTI equations can be represented as

$$V_{Pvert} = V_{P0}^{(V)} = V_{P0}\sqrt{1+2\epsilon} , \tag{24}$$

and the shear-wave vertical velocities are given by

$$V_{S^{\perp}vert} = V_{S0} , \tag{25}$$

$$V_{S^{\parallel}vert} = V_{S0}\sqrt{1+2\gamma} . \tag{26}$$

As discussed above, the difference between the vertical velocities of the waves $S^{\perp}$ and $S^{\parallel}$ makes it necessary to consider two different equivalent VTI models for $P - S^{\perp}$ and $S^{\parallel}$ propagation.

Note that if the vertical and horizontal P–wave velocities are equal to each other ($\epsilon = 0$), the phase and group velocities of the P– and $S^{\perp}$–waves in the symmetry plane are symmetric with respect to the 45-degree angle. As a result, in this case

$P - S^{\perp}$ velocities do not change if we rotate the symmetry axis by 90 degrees, and $\delta^{(V)}$ is equal to δ ($\epsilon^{(V)} = \epsilon = 0$). The same holds for the $S^{\parallel}$–wave if $\gamma = 0$, but this is the trivial case of a medium with no $S^{\parallel}$–wave velocity anisotropy.

The above expressions can be applied to describe wave propagation in HTI media using the known equations for vertical transverse isotropy, but in terms of the "generic" Thomsen parameters of the HTI model. Although the analogy between HTI and VTI media is limited to the single symmetry plane that contains the symmetry axis, the anisotropic coefficients of the equivalent VTI medium turn out to be responsible for the azimuthal dependence of NMO velocity as well.

NORMAL MOVEOUT FROM A HORIZONTAL REFLECTOR

NMO velocity in symmetry planes

First, let us consider the influence of anisotropy on the normal-moveout velocity for survey lines in the symmetry planes of an HTI layer (Figure 1). If the CMP line is perpendicular to the symmetry axis, the incident and reflected rays are confined to the isotropy plane, and the NMO velocities of each wave are just equal to the corresponding vertical velocities. Therefore, here I consider the second vertical symmetry plane that contains the symmetry axis (the "symmetry" plane).

The simplest way to obtain NMO velocities in the symmetry plane is to use the known NMO equations for vertical transverse isotropy (e.g., Thomsen, 1986) and the relations between the anisotropic parameters of HTI and VTI media given in the previous section. Alternatively, as shown in Appendix B, normal-moveout velocity can be obtained directly from the phase-velocity equations for horizontal transverse isotropy. Both approaches lead to the same expression for the NMO velocity of the P–wave valid for any strength of the anisotropy:

$$V_{\rm nmo}\,[P\text{—wave}] = V_{Pvert}\sqrt{1+2\delta^{(\rm V)}} = V_{Pvert}\sqrt{1+2\,\frac{\delta-2\epsilon\left(1+\frac{\epsilon}{f}\right)}{(1+2\epsilon)\left(1+\frac{2\epsilon}{f}\right)}}\,, \qquad (27)$$

in accordance with equation (1) and the analogy between VTI and HTI media outlined above; $\delta^{(\rm V)}$ is given by equation (20).

Equation (27) can be rewritten using the P–wave velocity in the symmetry direction V_{P0} as (Appendix B)

$$V_{\rm nmo}\,[P\text{—wave}] = V_{P0}\sqrt{1-\frac{2(\epsilon-\delta)}{1+\frac{2\epsilon}{f}}}\,. \qquad (28)$$

As discussed in the previous section, for $\epsilon = 0$ the P–wave phase and group velocity in the plane containing the symmetry axis are symmetric with respect to the

45-degree angle, and the NMO velocities in terms of δ and $\delta^{(V)}$ become identical to each other.

Note that the expressions (27) and (28) for P–wave NMO velocity include not only the P–wave horizontal velocity V_{P0} and the anisotropy parameters ϵ and δ, but also the shear-wave horizontal velocity V_{S0} (contained in the parameter f), which seem to contadict the conclusion by Tsvankin (1995b) about the small influence of V_{S0} on P–wave traveltimes. However, the velocity V_{S0} contributes just to the term quadratic in the anisotropic coefficients ϵ and δ and, therefore, has only a small impact on the P–wave NMO velocity.

For weak anisotropy ($\epsilon \ll 1$, $\delta \ll 1$), we can simplify equations (27) and (28) by retaining only the terms linear in ϵ and δ:

$$V_{\text{nmo}}(P\text{–wave}) \approx V_{P vert}\,(1+\delta^{(V)}) \approx V_{P0}\,(1+\delta-\epsilon)\,. \qquad (29)$$

Equation (29) coincides with the expression given by Sena [1991, equation (A-10)], who calls the NMO velocity the "skewed" moveout velocity. Note that the corresponding equation (12a) in Thomsen (1988) is in error.

Similarly, the exact NMO velocity for the wave $S^{\perp}$ is given by

$$V_{\text{nmo}}\,[S^{\perp}\text{–wave}] = V_{S0}\,\sqrt{1+2\sigma^{(V)}} = V_{S0}\,\sqrt{1+\frac{2\sigma}{1+\frac{2\epsilon}{f}}}\,, \qquad (30)$$

with $\sigma^{(V)}$ from equation (23). Since for the $S^{\perp}$–wave the velocities in the symmetry direction and in the perpendicular (isotropy) plane are identical, V_{S0} in equation (30) represents both the vertical and horizontal velocity.

As for P–waves, the $S^{\perp}$–wave NMO velocities in VTI and HTI media with the same values of the Thomsen parameters coincide with each other for $\epsilon = 0$. Furthermore, the phase and group velocity of the wave $S^{\perp}$ in any plane containing the symmetry axis are almost symmetric with respect to the 45-degree angle even for $\epsilon \neq 0$. Indeed, in the limit of weak anisotropy $\sigma^{(V)} = \sigma$, and equation (30) reduces to

$$V_{\text{nmo}}\,[S^{\perp}\text{–wave}] \approx V_{S0}\,(1+\sigma)\,. \qquad (31)$$

Equation (31) coincides with the weak-anisotropy approximation for the $S^{\perp}$–wave NMO velocity in VTI media [equation (2)] for any values of ϵ. If the $S^{\perp}$–wave velocity anisotropy is not weak, the NMO velocities of $S^{\perp}$ in VTI and HTI media with the same values of the Thomsen parameters are not equal but remain close to each other (in the symmetry plane). Note that equation (31) is equivalent to equation (12b) in Thomsen (1988) presented in a different form.

For the $S^{\parallel}$–wave, a 90-degree rotation of the symmetry axis is equivalent to interchanging the elliptical axes, and the NMO velocity remains equal to the horizontal shear-wave velocity:

$$V_{\text{nmo}}[S^{\parallel}\text{—wave}] = V_{S^{\parallel}vert}\sqrt{1+2\gamma^{(V)}} = V_{S0}\,. \tag{32}$$

Azimuthal dependence of NMO velocity

Normal-moveout velocity in symmetry planes of any anisotropic medium can be studied using the equation of Tsvankin (1995a) derived under the assumption that the phase and group velocities of the reflected waves lie in the incidence plane. For a survey line over an HTI medium that is neither parallel nor perpendicular to the symmetry axis (Figure 3), the phase-velocity vectors may deviate from the incidence plane, thus making this equation inaccurate. A more general expression for NMO velocity that fully honors the 3–D behavior of the phase- and group-velocity vectors in HTI media is obtained in Appendix A:

$$V_{\text{nmo}}^2 = V_{vert}^2\,\frac{1+\frac{1}{V}\frac{d^2V}{d\theta^2}}{1+\sin^2\alpha\left[\frac{1}{V}\frac{d^2V}{d\theta^2}\right]}\,, \tag{33}$$

where $V(\theta)$ is the phase velocity as a function of the phase angle; the phase velocity and its second derivative should be evaluated at the vertical phase (and group) direction. Equation (33) is valid for HTI models with any strength of the anisotropy and can be used for all three pure modes (P, $S^{\perp}$, $S^{\parallel}$).

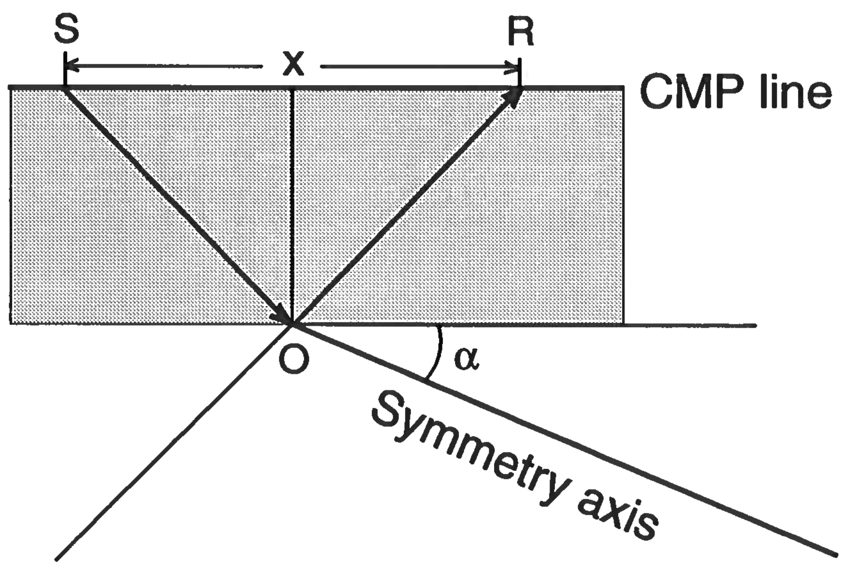

FIG. 3. Common-midpoint reflections over a transversely isotropic layer with a horizontal axis of symmetry. The symmetry axis makes the angle α with the survey (CMP) line. As shown in Appendix A, the incident and reflected rays of pure modes lie in the incidence (sagittal) plane, while the corresponding phase-velocity vectors may deviate from the plane.

Two special cases considered in the previous section correspond to the survey line parallel ($\alpha = 0$) and perpendicular ($\alpha = 90°$) to the symmetry axis. In the latter case ($\alpha = 90°$), the incident and reflected rays lie in the isotropy plane, and the NMO velocity from equation (33) becomes equal to the vertical velocity,

$$V_{\text{nmo}}(\alpha = 90°) = V_{vert}\,. \tag{34}$$

If the survey line is confined to the symmetry plane (i.e., is parallel to the symmetry axis) and $\alpha = 0$, equation (33) reduces to

$$V^2_{\text{nmo}}(\alpha = 0) = V^2_{vert}\left(1 + \frac{1}{V}\frac{d^2V}{d\theta^2}\right), \tag{35}$$

which is identical to NMO equation of Tsvankin (1995a) for the special case of a horizontal reflector. NMO velocity described by equation (35) has been discussed in detail for each wave type (P, $S^{\perp}$, $S^{\parallel}$) in the previous section.

The most interesting and somewhat surprising feature of equation (33) is that the influence of anisotropy on NMO velocity is absorbed by a single term that we will denote as A:

$$A = \frac{1}{V}\frac{d^2V}{d\theta^2} = \frac{V^2_{\text{nmo}}(\alpha = 0)}{V^2_{vert}} - 1\,. \tag{36}$$

Using equations (27), (30), and (32), we can identify A for different wave types as

$$A\,[P\text{—wave}] = 2\delta^{(\text{V})} = 2\,\frac{\delta - 2\epsilon\left(1 + \frac{\epsilon}{f}\right)}{(1 + 2\epsilon)\left(1 + \frac{2\epsilon}{f}\right)}, \tag{37}$$

$$A\,[S^{\perp}\text{—wave}] = 2\sigma^{(\text{V})} = 2\,\frac{\sigma}{1 + \frac{2\epsilon}{f}}, \tag{38}$$

$$A\,[S^{\parallel}\text{—wave}] = 2\gamma^{(\text{V})} = -2\,\frac{\gamma}{1 + 2\gamma}\,. \tag{39}$$

Equations (37)–(39) demonstrate that the azimuthal dependence of NMO velocity for horizontal transverse isotropy is governed by the Thomsen parameters of the equivalent VTI medium. In the weak-anisotropy approximation, the expressions for A can be linearized in the anisotropic parameters to give

$$A\,[P\text{—wave}] = 2\,(\delta - 2\epsilon)\,, \tag{40}$$

$$A\,[S^{\perp}\text{—wave}] = 2\,\sigma\,, \tag{41}$$

$$A\,[S^{\parallel}\text{—wave}] = -2\,\gamma\,, \tag{42}$$

while equation (33) for NMO velocity becomes

$$V_{\text{nmo}}^2 = V_{vert}^2 \left(1 + A \cos^2 \alpha\right). \tag{43}$$

Thus, normal-moveout velocity for horizontal transverse isotropy is a relatively simple function of three parameters: the vertical velocity, the azimuthal angle between the survey line and the symmetry axis, and the anisotropic term A.

INVERSION OF NMO VELOCITY FOR HORIZONTALLY-LAYERED MEDIA

The NMO equations given above can be used to invert the moveout velocities for the anisotropic parameters. One of the potential complications in this inversion is the distortions caused by nonhyperbolic moveout. Indeed, reflection moveout even in a homogeneous anisotropic medium is generally nonhyperbolic, and the moveout velocity on a finite-length spread may be different from the "zero-spread" NMO velocity (Hake et al., 1984). However, as shown by Tsvankin and Thomsen (1994) for horizontal reflectors beneath VTI media, deviations of P–wave moveout from hyperbolic for conventional spreadlengths (equal to the distance between the CMP and the reflector) are small. SV–wave moveout on these spreads is also close to hyperbolic for the most common, positive values of the difference $\epsilon - \delta$ (Tsvankin and Thomsen, 1994). Furthermore, the magnitude of nonhyperbolic moveout decreases with reflector dip (Anderson and Tsvankin, 1995). These conclusions can be extended to the vertical plane in HTI media that contains the symmetry axis by using the equivalence between VTI and HTI media. Note that the $S^{\parallel}$–wave moveout in this plane is purely hyperbolic, both for horizontal and dipping reflectors (Uren et al., 1990). Obviously, reflection moveouts of all three waves are purely hyperbolic on the survey line in the isotropy plane ($\alpha = 90°$). Estimation of the magnitude of nonhyperbolic moveout outside symmetry planes requires a numerical study.

Although the NMO equations derived in the previous section are valid for a single HTI layer, they can be applied in a straightforward fashion to certain types of more realistic vertically inhomogeneous models. Suppose the medium consists of a stack of vertically and horizontally transversely isotropic layers with the symmetry axis in HTI media pointing in either of two arbitrary but orthogonal directions. Then these directions will determine two vertical symmetry planes of this anisotropic model in which we can apply the generalized Dix equation presented by Alkhalifah and Tsvankin (1995). In the case of horizontal reflectors, this equation reduces to the standard Dix (1955) formula that allows one to obtain the NMO velocity for any layer from the NMO velocities for the reflections from the top and bottom of this layer. Thus, we can recover the single-layer NMO velocities in symmetry planes of VTI–HTI stratified media by the conventional Dix differentiation procedure. Although the Dix equation does not work exactly outside the symmetry planes, it can still be expected to provide a good approximation for weak and moderate azimuthal anisotropy.

As mentioned above, in the case of a horizontal symmetry axis we can exploit the azimuthal dependence of reflection data by using NMO velocities measured on survey lines with different orientation (e.g., using 3–D surveys). Due to the fact that the influence of anisotropy in NMO equation (33) is concentrated in the single parameter A, it is sufficient to have three measurements of NMO velocity at different azimuthal angles for any wave type to recover the three unknowns, including the orientation of the symmetry axis (α, V_{vert} and A). Although $A/2$ is equal to the parameters of the equivalent VTI model ($\delta^{(V)}$, $\sigma^{(V)}$, or $\gamma^{(V)}$ depending on the wave type), such an algorithm cannot be devised for vertical transverse isotropy; obviously, in VTI media moveout velocities do not vary with azimuth. As mentioned above, NMO velocities in VTI media are not sufficient to invert for the vertical velocities and anisotropic coefficients, even if both P and shear data are used.

If the vertical velocity of one of the waves has been determined (say, $V_{P_{vert}}$ from well logs or check shots), then the parameter A for the P–wave and the angle α can be obtained from just two P–wave NMO velocities measured at different azimuths. Then a single NMO velocity for any other mode (say, $S^{\perp}$) is sufficient to obtain the anisotropic parameter A for this wave since the vertical velocity ($V_{S^{\perp}vert}$) in this case can be found from $V_{P_{vert}}$ and the vertical P and $S^{\perp}$ traveltimes.

In another scenario, the orientation of the symmetry axis may be known, which is often the case for HTI reservoirs with the anisotropy caused by vertical cracks. For instance, the crack orientation that determines the direction of the symmetry axis can be obtained from shear-wave VSP's. Then the NMO velocities in two azimuthal directions are sufficient to invert for the vertical velocity and the coefficient A. For instance, we may be able to obtain the vertical velocities by performing moveout analysis on the survey line normal to the symmetry axis [equation (34)]. Then the NMO velocities on a line with any other orientation make it possible to find the anisotropic parameter A. Finally, if both the axis orientation and the vertical velocity are known, a single value of NMO velocity (for instance, on the line parallel to the symmetry axis) can be inverted for the parameter A.

Determination of Thomsen parameters

To carry out seismic processing outside symmetry planes, we need to know the "generic" Thomsen parameters of the model (ϵ, δ, γ) that determine the phase and group velocities of all three waves as functions of the angle with the symmetry axis. Hence, the next question to be answered is what information about ϵ, δ, and γ can be obtained from the vertical velocities and the values of A recovered from moveout data.

P–wave processing requires knowledge of the velocity V_{P0} and the anisotropic parameters ϵ and δ. As shown above, P–wave NMO data can yield the vertical velocity $V_{P_{vert}}$ and $\delta^{(V)} = A/2$ given by [equations (20) or (37)]:

$$\delta^{(V)} = \frac{\delta - 2\epsilon\left(1 + \frac{\epsilon}{f}\right)}{(1 + 2\epsilon)\left(1 + \frac{2\epsilon}{f}\right)} \,. \tag{44}$$

For weak anisotropy, $\delta^{(V)} \approx \delta - 2\,\epsilon$. Thus, P–wave data alone enable us to find an anisotropic coefficient close to $\delta - 2\,\epsilon$. In the special case of $\epsilon = 0$, discussed in more detail below, δ is simply equal to $\delta^{(V)}$, and $V_{P0} = V_{Pvert}$. In another special case of the known V_{P0} (e.g., from head waves or cross-hole tomography), the parameter ϵ can be found using the vertical P–wave velocity. Alternatively, the presence of dipping events makes it possible to obtain $\epsilon^{(V)}$ (and ϵ) from P–wave NMO velocities. In this case, the symmetry-direction velocity V_{P0} can be found from V_{Pvert} and ϵ using equation (24). Once ϵ has been determined, $\delta^{(V)}$ [equation (44)] is sufficient to resolve δ given an approximate value of the V_{P0}/V_{S0} ratio (this ratio contributes only to the terms quadratic in the anisotropic parameters).

By inverting $S^{\perp}$–wave moveout data, we can obtain the vertical (and horizontal) $S^{\perp}$–wave velocity $V_{S^{\perp}vert} = V_{S0}$ and the anisotropic term $\sigma^{(V)} = A/2$ [equations (23) or (38)]:

$$\sigma^{(V)} = \frac{\sigma}{1 + \frac{2\epsilon}{f}} \,. \tag{45}$$

In the limit of weak anisotropy,

$$\sigma^{(V)} \approx \sigma = \left(\frac{V_{P0}}{V_{S0}}\right)^2 (\epsilon - \delta) \,. \tag{46}$$

Assuming that $S^{\perp}$–wave anisotropy is moderate and we have a good estimate of the V_{P0}/V_{S0} ratio, we can use equation (46) to evaluate the difference $\epsilon - \delta$.

Although V_{vert} and A are the only two medium parameters (except for the axis orientation) that can be obtained for any single wave type, we can combine, for instance, P and $S^{\perp}$–waves to resolve the anisotropic coefficients individually. In fact, it is sufficient to find one of the vertical velocities since the second velocity can then be recovered from the ratio of the vertical P and $S^{\perp}$ traveltimes. The coefficients $\delta^{(V)}$ and $\sigma^{(V)}$ [equations (44) and (45)], along with the vertical velocities V_{Pvert} and $V_{S^{\perp}vert} = V_{S0}$, are sufficient to resolve all four parameters responsible for $P - S^{\perp}$ propagation (V_{P0}, V_{S0}, ϵ, and δ), as well as the reflector depth. Since V_{S0} is already known, all we have to do is to substitute

$$V_{P0} = \frac{V_{Pvert}}{\sqrt{1 + 2\,\epsilon}}$$

into equations (44) and (45) and solve them for ϵ and δ. It is clear that this inversion is stable because $\delta^{(V)}$ is close to $\delta - 2\,\epsilon$, while $\sigma^{(V)}$ provides an estimate of $\epsilon - \delta$.

The fifth parameter, γ, is usually obtained for HTI media directly from the fractional difference between the vertical $S^{\perp}$ and $S^{\parallel}$ velocities using the traveltimes of split shear waves at vertical incidence (e.g., Crampin, 1985; Thomsen, 1988):

$$\gamma = \frac{1}{2} \left(\frac{V^2_{S^{\parallel} vert}}{V^2_{S^{\perp} vert}} - 1 \right) . \tag{47}$$

The above results suggest an alternative way of recovering γ using just $S^{\parallel}$ moveout data. The azimuthal dependence of the $S^{\parallel}$–wave NMO velocities can be inverted for $\gamma^{(V)} = A/2$ and, consequently, for γ. If the symmetry direction is known, this inversion requires the $S^{\parallel}$–wave NMO velocities on two lines with different orientation. In the simplest case of the lines parallel and perpendicular to the symmetry axis,

$$\gamma = \frac{1}{2} \left(\frac{V^2_{\text{nmo}}(\alpha = 90^\circ)}{V^2_{\text{nmo}}(\alpha = 0)} - 1 \right) . \tag{48}$$

Note that if $S^{\parallel}$ and $S^{\perp}$ data are available, the ratio of the vertical shear-wave velocities can be obtained not only from the vertical traveltimes, but also from the respective NMO velocities. Then, γ can be calculated from equation (47).

Inversion for crack density

It is believed that the most common physical reason for horizontal transverse isotropy is the presence of parallel vertical cracks (fractures) embedded in an isotropic medium (Crampin, 1985). Thus, the question to be answered next is whether we can obtain reliable information about the properties of crack systems from moveout data.

The parameter of most interest in the characterization of fractured reservoirs is the crack density (D_c) that is proportional to the product of the number of cracks per unit volume and their mean cubed diameter. Although all three anisotropic coefficients (ϵ, δ, γ) are proportional to D_c, the parameter most directly related to crack density is γ. For parallel, penny-shaped cracks distributed in a porous isotropic rock, γ is given by (Thomsen, 1995)

$$\gamma = \frac{8}{3} \frac{1 - P}{2 - P} D_c , \tag{49}$$

where P is the Poisson's ratio of the dry isotropic porous medium.

It is easy to see that for plausible values of the Poisson's ratio the coefficient $8(1 - P)/[3(2 - P)]$ is close to unity, and $\gamma \approx D_c$. Therefore, measurements of γ provide a good direct estimate of the crack density. The most conventional way to recover γ is to use the traveltimes of the split shear waves at vertical incidence, if both shear modes are recorded. As suggested above, γ can also be obtained from $S^{\parallel}$–wave moveout data; the wave $S^{\perp}$ in this case is not needed at all.

The relation between ϵ, δ, and the crack density is complicated by such quantities as the incompressibility of the solid grains and the fluid in the cracks, as well as by the so-called "fluid influence factor" (Thomsen, 1995). This makes the inversion of ϵ and δ for the crack density an ambiguous procedure, unless detailed information about the physical properties of the rock is available. Since the moveouts of P and $S^{\perp}$–waves are controlled by ϵ and δ while being independent of γ, it seems that there is no straighforward way to invert NMO velocities of the P– and $S^{\perp}$–waves for the crack density.

However, the main difference between general transverse isotropy and TI media due to a system of thin parallel cracks is that in the latter case elastic constants satisfy the following equation that reduces the number of independent stiffness coefficients from five to four (Schoenberg and Sayers, 1995; Thomsen, 1995):

$$c_{11}c_{33} - c_{13}^2 = 2c_{66}\left(c_{13} + c_{33}\right), \tag{50}$$

with the cracks perpendicular to the x_3 axis. Replacing the stiffness coefficients in equation (50) by the Thomsen parameters from equations (5)–(9) yields

$$D_c \approx \gamma = \frac{V_{P0}^2}{2\,V_{S0}^2} \, \frac{\frac{\epsilon}{f} - \delta}{\left(1 + \sqrt{1 + \frac{2\delta}{f}}\right)}, \tag{51}$$

where f is given by equation (22). Other constraints for such a medium require that $\epsilon \geq 0$ and $\gamma \geq 0$.

Replacing the generic Thomsen parameters with the coefficients of the equivalent VTI medium using equations (19), (20), (24), and (25), we find

$$D_c \approx \gamma = \frac{V_{Pvert}^2}{2\,V_{S^{\perp}vert}^2} \, \frac{\epsilon^{(V)}\left[2 - 1/f^{(V)}\right] - \delta^{(V)}}{1 + 2\epsilon^{(V)}/f^{(V)} + \sqrt{1 + 2\,\delta^{(V)}/f^{(V)}}}, \tag{52}$$

with $f^{(V)} \equiv 1 - V_{S^{\perp}vert}^2/V_{Pvert}^2$.

Equation (52) expresses γ and the crack density through the parameters of the equivalent VTI medium that we can determine from $P - S^{\perp}$ moveout data. If $\epsilon^{(V)}$, $\delta^{(V)}$, and $V_{Pvert}/V_{S^{\perp}vert}$ have been found from NMO velocities of the P– and $S^{\perp}$–waves using the algorithm outlined above, the parameter γ and, consequently, the crack density D_c can be calculated from equation (52).

P–wave moveout data, combined with an approximate value of the ratio of the vertical velocities ($V_{Pvert}/V_{S^{\perp}vert}$), are sufficient to estimate the crack density if the P–wave velocity along the symmetry direction (V_{P0}) is known. Then V_{Pvert} and $\delta^{(V)}$ can be found from the P–wave NMO velocity, while $\epsilon^{(V)}$ is given by

$$\epsilon^{(V)} = \frac{1}{2}\left(\frac{V_{P0}^2}{V_{Pvert}^2} - 1\right).$$

Below, I suggest an alternative way of estimating $\epsilon^{(V)}$ using the dip-dependence of P–wave NMO velocity.

The inversion of P–wave data for the crack density becomes particularly simple in the special case of the equal vertical and horizontal (along the symmetry direction) velocities of the P–wave ($\epsilon = \epsilon^{(V)} = 0$) that corresponds to negligible equant porosity and "very thin" cracks (for quantitative estimates, see Thomsen, 1995). Such a model may be typical, for instance, for fractured coals that are of primary importance in methane production. If $\epsilon^{(V)} = 0$, equation (52) reduces to

$$D_c \approx \gamma = \frac{V^2_{Pvert}}{2\,V^2_{S^\perp vert}} \frac{-\delta^{(V)}}{1 + \sqrt{1 + 2\,\delta^{(V)}/f^{(V)}}} \,. \tag{53}$$

Since the parameter $\delta^{(V)}$ can be obtained from P–wave NMO velocity, P–wave data in this case are sufficient to obtain the crack density provided an approximate value of the $V_{Pvert}/V_{S^\perp vert}$ ratio is known. The combination of P and $S^\perp$ data may be necessary only to get a better estimate of $V_{Pvert}/V_{S^\perp vert}$, but γ, as determined by equation (53), is not too sensitive to realistic errors in this parameter. The assumption about $\epsilon = \epsilon^{(V)} = 0$, however, cannot be verified unless $\epsilon^{(V)}$ is recovered.

In the limit of weak anisotropy, equation (52) becomes

$$D_c \approx \gamma = \frac{V^2_{Pvert}}{4\,V^2_{S^\perp vert}} \left[\epsilon^{(V)} \left(2 - \frac{1}{f^{(V)}}\right) - \delta^{(V)}\right] , \tag{54}$$

If $V_{Pvert}/V_{S^\perp vert} = 2$, equation (54) further simplifies to

$$D_c \approx \gamma = 0.67\epsilon^{(V)} - \delta^{(V)} \,. \tag{55}$$

Equation (55) suggests that for weakly anisotropic HTI models the parameter γ (and the crack density) is close to the difference between $\epsilon^{(V)}$ and $\delta^{(V)}$. As demonstrated below, this difference can be evaluated using the dip-dependence of the P–wave NMO velocity. Also, γ can be roughly estimated using just $S^\perp$ NMO velocity that provides the parameter $\sigma^{(V)} = V^2_{Pvert}/V^2_{S^\perp vert}\,(\epsilon^{(V)} - \delta^{(V)})$.

In terms of the generic Thomsen parameters, the weak-anisotropy approximation for γ is given by [see equation (51)]

$$D_c \approx \gamma = \frac{V^2_{P0}}{4\,V^2_{S0}} \left(\frac{\epsilon}{f} - \delta\right) , \tag{56}$$

in agreement with Thomsen (1995). Since γ is non-negative, it is clear from equation (56) that the difference between ϵ and δ for TI media due to parallel cracks is typically positive.

NMO VELOCITY FROM DIPPING REFLECTORS

Normal-moveout velocity from both horizontal and dipping reflectors in symmetry planes of any homogeneous anisotropic medium, including horizontal transverse isotropy, can be studied using the following equation given by Tsvankin (1995a):

$$V_{\rm nmo}(\phi) = \frac{V(\phi)}{\cos\phi} \frac{\sqrt{1 + \frac{1}{V(\phi)} \frac{d^2V}{d\theta^2}}}{1 - \frac{\tan\phi}{V(\phi)} \frac{dV}{d\theta}}, \tag{57}$$

where V is phase velocity, θ is the phase angle with vertical, and ϕ is the dip angle of the reflector; the derivatives of phase velocity are evaluated at the dip ϕ. Equation (57) is strictly valid for 2-D wave propagation, with phase and group velocities of the reflected waves confined to the incidence plane. This implies that the incidence plane should represent both a symmetry plane of the medium and the dip plane of the reflector.

Equation (57) is applied below to the vertical symmetry plane of HTI media that contains the symmetry axis (here called simply the symmetry plane; Figure 1). The dip-moveout signature in the isotropy plane is trivial because velocity is independent of propagation angle. For any other survey (CMP) line making an arbitrary angle with the symmetry axis, equation (57) can be used only for weak azimuthal anisotropy. To comply with the assumptions behind equation (57), the incidence plane is taken to be the dip plane of the reflector, i.e., the strike of the reflector is perpendicular to the symmetry axis.

For homogeneous, isotropic media equation (57) reduces to the simple cosine-of-dip relationship (Levin, 1971):

$$V_{\rm nmo}(\phi) = \frac{V_{\rm nmo}(0)}{\cos\phi}. \tag{58}$$

Velocity variations with angle in anisotropic media [represented by the derivatives in equation (57)] lead to deviations from cosine-of-dip formula (58). Therefore, NMO velocities from dipping reflectors can provide useful information about anisotropy for the inversion procedure (Alkhalifah and Tsvankin, 1995). Description of dip-dependent NMO velocity is also important in developing dip-moveout (DMO) algorithms, as well as other seismic processing methods for anisotropic media (Anderson and Tsvankin, 1995).

NMO velocity as a function of dip angle

The equivalence between vertical and horizontal transverse isotropy discussed above implies that the results for VTI media can be directly used in the symmetry plane of HTI media that contains the symmetry axis. Here, however, we need to study the dip-dependence of NMO velocity for typical values of the generic Thomsen parameters ϵ, δ, and γ.

To understand the influence of the anisotropic parameters on NMO velocity, it is convenient to apply the weak-anisotropy approximation to equation (57). In the limit of weak anisotropy, P–wave NMO velocity for vertical transverse isotropy can be expressed through phase velocity as (Tsvankin, 1995a)

$$V_{\text{nmo}}(\phi)\cos(\phi) = V_{\text{nmo}}(0)\left\{\frac{V_P(\phi)}{V_{Pvert}}\left[1+2(\epsilon^{(\mathrm{V})}-\delta^{(\mathrm{V})})\sin^2\phi\,(1+2\cos^2\phi)\right]\right\}. \tag{59}$$

Equation (59) is structured to show the deviation of the NMO velocity from the isotropic cosine-of-dip expression (58). Using equations (19) and (20), we find that for weak anisotropy

$$\epsilon^{(\mathrm{V})} - \delta^{(\mathrm{V})} = \epsilon - \delta\,,$$

and equation (59) retains the same form if expressed through ϵ and δ. However, the angle dependence of the P–wave phase velocity $V_P(\phi)$ in equation (59) is substantially different in models with horizontal and vertical orientations of the symmetry axis. Indeed, in HTI media the horizontal velocity is smaller than the vertical velocity ($\epsilon^{(\mathrm{V})} < 0$), while in VTI media the opposite is true.

After being fully linearized in the anisotropic parameters, equation (59) becomes (Tsvankin, 1995a)

$$\frac{V_{\text{nmo}}(\phi)\cos\phi}{V_{\text{nmo}}(0)} = 1 + \delta^{(\mathrm{V})}\sin^2\phi + 3(\epsilon^{(\mathrm{V})}-\delta^{(\mathrm{V})})\sin^2\phi\,(2-\sin^2\phi)\,. \tag{60}$$

Substituting the expressions for $\epsilon^{(\mathrm{V})}$ and $\delta^{(\mathrm{V})}$ from equations (19) and (20), we find an equivalent weak-anisotropy approximation for horizontal transverse isotropy:

$$\frac{V_{\text{nmo}}(\phi)\cos\phi}{V_{\text{nmo}}(0)} = 1 - \delta\sin^2\phi + 3(\epsilon-\delta)\sin^2\phi\,(\frac{4}{3}-\sin^2\phi)\,. \tag{61}$$

Again, equations for VTI (60) and HTI (61) media are similar to each other, with the dip-dependence of NMO velocity for both models mostly influenced by the second anisotropic term that contains the difference $\epsilon - \delta$. As discussed above, $\epsilon - \delta$ is typically positive for HTI media (as for vertical transverse isotropy), and we can expect the cosine-of-dip corrected NMO velocity to increase with dip. However, since the trigonometric coefficient multiplied with $\epsilon - \delta$ is smaller for HTI than for VTI media, the individual contribution of δ in equation (61) is more pronounced than in equation (60). Also note that the signs of the δ term in equations (60) and (61) are opposite. These conclusions are further illustrated by the numerical examples shown in Figures 4 and 5. For typical positive values of $\epsilon - \delta$ the cosine-of-dip corrected NMO velocity increases with dip much slower than in VTI media with the same values of the anisotropic parameters (Tsvankin, 1995a). Therefore, for models with the same positive $\epsilon - \delta$, the accuracy of the isotropic cosine-of-dip relationship is higher in HTI media than for vertical transverse isotropy. Although the weak-anisotropy approximation in Figure 4 is close to the exact NMO velocity for small and moderate values of ϵ and δ, the weak-anisotropy formula tends to overstate the influence of the anisotropy.

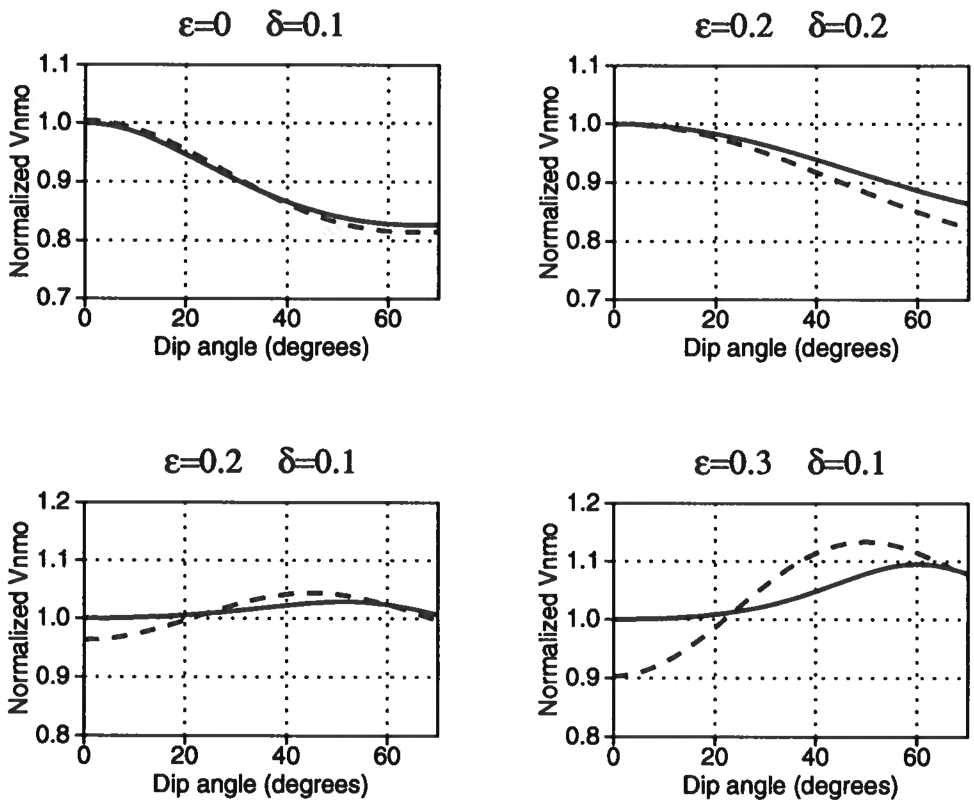

FIG. 4. The dip-dependence of P-wave normal-moveout velocity in the symmetry plane of HTI media that contains the symmetry axis. NMO velocity is multiplied with the cosine of the dip angle and divided by the exact $V_{\rm nmo}(0)$ to show the error in the isotropic equation (58) caused by the anisotropy. The solid curve is the exact NMO velocity [equation (57)]; the dashed curve is the weak-anisotropy approximation from equation (61).

Also, in contrast with the results for vertical transverse isotropy described by Tsvankin (1995a), the dependence of P-wave NMO velocity on the dip angle is much less controlled by the difference between ϵ and δ (Figure 5). This result was explained above in terms of the weak-anisotropy approximation; the difference between the exact and weak-anisotropy NMO velocity leads to further separation between the curves corresponding to models with the same value of $\epsilon - \delta$.

For the wave $S^{\perp}$, the weak-anisotropy approximation can be obtained from equation (61) by making the substitutions $\delta = \sigma$ and $\epsilon = 0$:

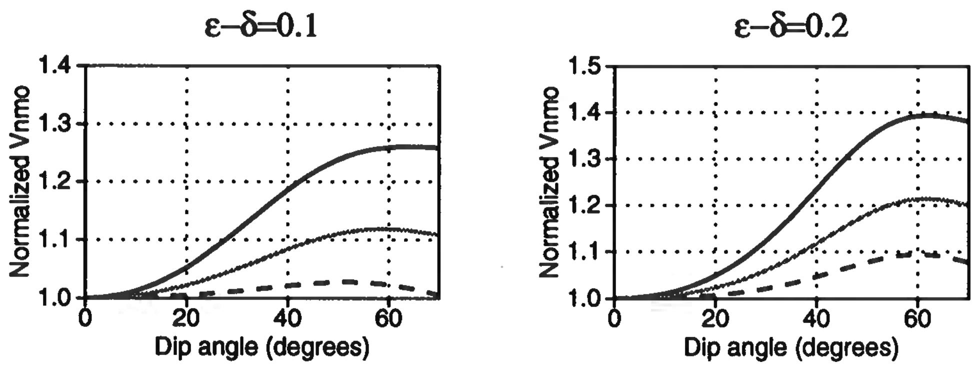

FIG. 5. Cosine-of-dip corrected P–wave NMO velocity calculated from equation (57). The curves on the left plot correspond to models with ϵ–δ=0.1: ϵ=0, δ=–0.1 (black curve); ϵ=0.1, δ=0 (gray curve); ϵ=0.2, δ=0.1 (dashed curve). On the right plot, ϵ–δ=0.2: ϵ=0.1, δ=–0.1 (black); ϵ=0.2, δ=0 (gray); ϵ=0.3, δ=0.1 (dashed).

$$\frac{V_{\mathrm{nmo}}(\phi)\cos\phi}{V_{\mathrm{nmo}}(0)}[S^{\perp}] = 1 - 5\sigma\sin^2\phi + 3\sigma\,\sin^4\phi\,. \tag{62}$$

Since σ is typically positive (both for VTI and HTI media), we can expect the cosine-of-dip corrected $S^{\perp}$–wave NMO velocity to decrease with dip.

In the case of the wave $S^{\parallel}$, the 90-degree rotation of the axis is equivalent to interchanging the elliptical axes, and the general NMO equation for elliptical anisotropy (Tsvankin, 1995a) holds in HTI media:

$$\frac{V_{\mathrm{nmo}}(\phi)\cos\phi}{V_{\mathrm{nmo}}(0)}[S^{\parallel}] = \frac{V_{S^{\parallel}}(\phi)}{V_{S^{\parallel}vert}}\,, \tag{63}$$

Equation (63) shows that for elliptical anisotropy the error in the cosine-of-dip dependence is determined directly by the phase-velocity variation. Substituting the phase-velocity function $V_{S^{\parallel}}(\phi)$ for a horizontal symmetry axis, we obtain

$$\frac{V_{\mathrm{nmo}}(\phi)\cos\phi}{V_{\mathrm{nmo}}(0)}[S^{\parallel}] = \sqrt{1 - \frac{2\gamma\sin^2\phi}{1+2\gamma}}\,, \tag{64}$$

Note that the dip-dependence of the $S^{\parallel}$–wave NMO velocity is exactly the same as that for the P–wave in elliptically anisotropic media (e.g., the model with $\epsilon = \delta = 0.2$ in Figure 4).

Parameter η and time processing in HTI media

For purposes of seismic processing it is more convenient to treat NMO velocity as a function of the ray parameter corresponding to zero-offset reflection. Then, as shown by Alkhalifah and Tsvankin (1995), the P–wave NMO velocity in VTI media is governed by just two parameters: the zero-dip NMO velocity $V_{\rm nmo}(0)$ [equation (1)] and the anisotropic coefficient $\eta^{(V)}$:

$$\eta^{(V)} = \frac{\epsilon^{(V)} - \delta^{(V)}}{1 + 2\delta^{(V)}} . \tag{65}$$

Both parameters ($V_{\rm nmo}(0)$ and $\eta^{(V)}$) can be reliably recovered from P–wave surface data using NMO velocities and ray parameters for two distinctly different dips. In VTI media, the parameters $V_{\rm nmo}(0)$ and $\eta^{(V)}$ are sufficient to perform all time-related processing steps including NMO correction, dip-moveout removal, prestack and poststack time migration (Alkhalifah and Tsvankin, 1995).

In essence, $\eta^{(V)}$ is responsible for the influence of transverse isotropy on P–wave NMO velocity and time-related processing in general. For elliptical anisotropy, $\eta^{(V)} = 0$, and NMO equation (57) reduces to the well-known expression valid for isotropic media:

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1 - p^2 V_{\rm nmo}^2(0)}} , \tag{66}$$

p is the ray parameter. The contribution of anisotropy in equation (66) is hidden in the values of the zero-dip NMO velocity $V_{\rm nmo}(0)$ and the ray parameter p. All isotropic time-processing methods remain valid for elliptical models, irrespective of the strength of velocity anisotropy.

Although wave propagation in the symmetry plane of HTI media that contains the symmetry axis can be described using VTI equations, the parameters of the equivalent VTI medium are different from those conventionally used for vertical transverse isotropy. For instance, since in HTI media the vertical P–wave velocity is *higher* than the horizontal velocity, the parameter $\epsilon^{(V)}$ of the equivalent VTI medium is *negative*, an extremely unusual case for vertical transverse isotropy.

Therefore, we have to check whether the conclusions by Alkhalifah and Tsvankin (1995) hold for the uncommon VTI models corresponding to HTI media with typical values of ϵ and δ. The parameter $\eta^{(V)}$ can be expressed through ϵ and δ using equations (19) and (20):

$$\eta^{(V)} = \frac{\epsilon - \delta}{1 + 2\delta} \, \frac{1}{1 - \frac{2\epsilon(1-1/f)}{1+2\delta}} = \eta \, \frac{1}{1 - \frac{2\epsilon(1-1/f)}{1+2\delta}} . \tag{67}$$

Figure 6a shows that P–wave NMO velocities for HTI models that have the same value of $\eta^{(V)}$ practically coincide with each other. This proves that the P–wave NMO velocity as a function of the ray parameter is entirely controlled by the zero-dip value $V_{\rm nmo}(0)$ and the parameter $\eta^{(V)}$, whether the medium has a vertical or horizontal symmetry axis. Also, as demonstrated by Figure 6b, the resolution in η

is high enough for a stable recovery of this parameter from the dip-dependence of P–wave NMO velocity. We conclude that time-related processing of P–wave data in the symmetry plane that contains the symmetry axis can be carried out using the algorithms developed for vertical transverse isotropy (e.g., Alkhalifah and Tsvankin, 1995).

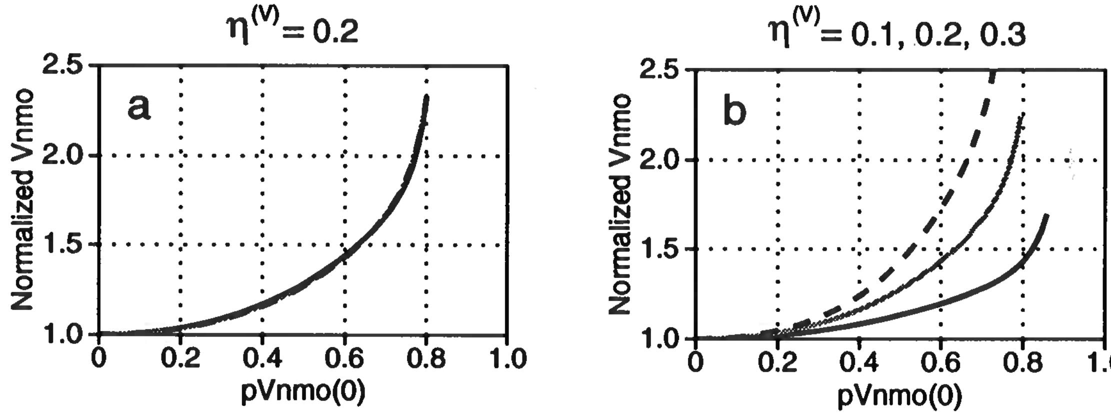

FIG. 6. P–wave normal-moveout velocity in the symmetry plane of HTI media calculated from equation (57) and normalized by the expression for isotropic media (66). The dip angles range between 0 and 70 degrees. (a) models with the same $\eta^{(V)}$=0.2: ϵ=0.1, δ=–0.0838 (solid); ϵ=0.2, δ=–0.0248 (gray); ϵ=0.3, δ=0.0343 (dashed) – the curves practically coincide with each other. The ratio V_{S0}/V_{P0}=0.55. (b) models with different $\eta^{(V)}$: $\eta^{(V)}$=0.1 (solid); $\eta^{(V)}$=0.2 (gray); $\eta^{(V)}$=0.3 (dashed).

Equation (67) indicates that despite the significant differences in the values of the Thomsen parameters for HTI media and the equivalent VTI media, the parameter η for both models remains almost the same. Indeed, Figure 7 shows the P–wave NMO-velocity curves for HTI models with a fixed value of $\eta = 0.2$ instead of the fixed $\eta^{(V)}$ in Figure 6. The corresponding $\eta^{(V)}$ changes from 0.182 to 0.163, which causes some separation between the curves in Figure 7a. Nevertheless, the NMO velocities for models with the same η remain sufficiently close to each other.

Thus, if we apply VTI inversion algorithms to the dip-dependence of P–wave NMO velocity in the symmetry plane of HTI media that contains the symmetry axis, we get the value of $\eta^{(V)} \approx \eta$. Alternatively, the parameter $\eta^{(V)}$ can also be obtained from the P–wave velocity in the symmetry direction (that may be known from head waves and/or cross-hole tomography) and the zero-dip NMO velocity without using dipping events (Alkhalifah and Tsvankin, 1995).

The value of $\eta^{(V)}$ adds new information to the inversion procedure described in the previous section and makes it possible to estimate the crack density just from P–wave moveout data. As discussed above, the P–wave vertical velocity and the parameter $\delta^{(V)}$ can be found using the P–wave NMO velocity from horizontal reflectors. The presence of dipping events makes it possible to recover $\epsilon^{(V)}$ from the parameter $\eta^{(V)}$. Then, given a rough estimate of the ratio of the P-to-$S^{\perp}$ vertical velocities, the crack

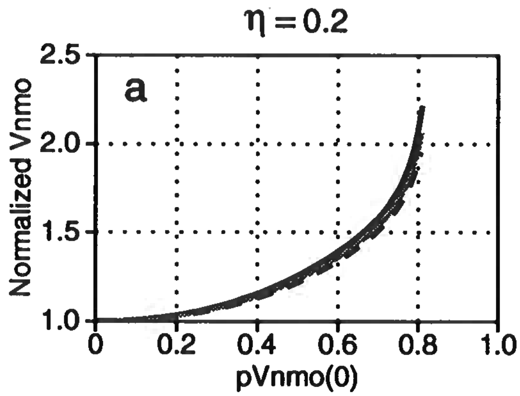

FIG. 7. P–wave moveout velocity calculated from equation (57) and normalized by the expression for isotropic media (66). The dip angles range between 0 and 70 degrees. (a) models with the same η=0.2: ϵ=0.1, δ=–0.071 (solid); ϵ=0.2, δ=0 (gray); ϵ=0.3, δ=0.071 (dashed). (b) models with different η: η=0.1 (solid); η=0.2 (gray); η=0.3 (dashed).

density can be calculated from equation (52). Another way to determine $\delta^{(V)}$ and $\epsilon^{(V)}$ is to combine the zero-dip NMO velocity and parameter $\eta^{(V)}$ obtained from P–wave data with the $S^{\perp}$–wave (or $P - S^{\perp}$–wave) NMO velocity from a horizontal reflector.

Even if no other data are available, just the value of $\eta^{(V)}$ can be used to make a crude estimate of the crack density. First of all, in the special case of $\epsilon = \epsilon^{(V)} = 0$, the parameter $\eta^{(V)}$ is sufficient to obtain $\delta^{(V)}$ and compute the crack density from equation (53). In the more general case of non-zero ϵ, we can represent γ in the weak-anisotropy approximation as [equation (55)]

$$D_c \approx \gamma = \frac{V_{Pvert}^2}{4\,V_{S^{\perp}vert}^2} \left[\epsilon^{(V)}\left(2 - \frac{1}{f^{(V)}}\right) - \delta^{(V)}\right] \approx 0.67\epsilon^{(V)} - \delta^{(V)}\,,$$

which is close to $\eta^{(V)} \approx \epsilon^{(V)} - \delta^{(V)}$. Obviously, the accuracy of such an estimate can be acceptable only for relatively small values of $\delta^{(V)}$ and, especially, $\epsilon^{(V)}$. Also, the term $2 - 1/f^{(V)}$ becomes closer to unity for higher $V_{Pvert}/V_{S^{\perp}vert}$ ratios. If $\delta^{(V)}$ is known to be small, the conversion of $\eta^{(V)}$ into the crack density can be improved by multiplying $\eta^{(V)}$ with $(2 - 1/f^{(V)})$:

$$D_c \approx \gamma\,[\text{small } \delta] = \epsilon^{(V)}\left(2 - \frac{1}{f^{(V)}}\right) = \eta^{(V)}\left(2 - \frac{1}{f^{(V)}}\right)\,.$$

This approximate way of estimating the crack density works better for substantially different $\epsilon^{(V)}$ and $\delta^{(V)}$ (relatively large $\eta^{(V)}$) than for media close to elliptically anisotropic with almost equal values of $\epsilon^{(V)}$ and $\delta^{(V)}$.

DISCUSSION AND CONCLUSIONS

Horizontal transverse isotropy is usually caused by the presence of vertical cracks (fractures) in an otherwise isotropic matrix. The parameter of crack systems of most interest in exploration is the crack density that is close to the fractional difference between the velocities of split shear waves at vertical incidence (Thomsen's coefficient γ). The shear-wave methods, developed extensively during the last decade, are designed to obtain γ directly from the shear-wave traveltimes and reflection amplitudes. This technology, however, has drawbacks associated with the cost of multicomponent surveys and the need to acquire high-quality shear data suitable for reliable polarization analysis. Also, shear-wave splitting yields an estimate of a single anisotropic parameter (γ), while the processing of P–wave data in HTI media requires knowledge of the other two coefficients (ϵ and δ).

Here, I have suggested several ways of estimating the anisotropic parameters by inverting normal-moveout information. The methodology is based on a new exact equation for NMO velocities from horizontal reflectors that is valid for any direction of the survey line with respect to the axis of symmetry. The influence of anisotropy on the azimuthally-dependent NMO velocity is absorbed by a single parameter responsible for normal moveout in the plane that contains the symmetry axis. The other two parameters in the NMO equation include the vertical velocity and the angle between the symmetry plane and the survey line. Therefore, three moveout measurements at different azimuthal angles can be inverted for the three unknowns or, in the case when the vertical velocity is known, two NMO velocities can be used to obtain the orientation of the symmetry axis and the effective anisotropic parameter.

This algorithm opens up the possibility of gaining information about the true vertical velocity (hence the reflector depth) and the principal directions of the anisotropy from 3–D P–wave surveys without using converted and shear modes. On the other hand, if shear data are available, the symmetry direction can be determined from S–wave polarizations, which simplifies the inversion of P–wave data for the vertical velocity and anisotropy. In general, it is highly beneficial to combine different types of data, such as moveout velocities, amplitudes (e.g., the azimuthal dependence of AVO response), and polarizations of P and shear waves.

The simplest way of inverting normal-moveout velocities for the crack density is to use the SH–wave NMO velocity on the lines parallel and perpendicular to the symmetry axis. However, it is also possible to infer the crack density from P– and $S^{\perp}$–wave moveout data. By using a constraint on the elastic constants of a medium with a system of parallel cracks, the crack density can be related to the parameters that can be obtained from P–wave and $S^{\perp}$–wave NMO velocities. P–wave normal moveout from horizontal reflectors is sufficient to estimate the crack density only in the special case of $\epsilon = 0$ that corresponds to negligible equant porosity and "very thin" cracks. This model may be relevant for coalbed methane plays with the production from low-porosity fractured coals. In the more general case of non-zero ϵ, P–wave NMO velocity from horizontal reflectors can be supplemented with moveout from dipping events or the velocity in the symmetry direction (obtained from cross-hole tomography or head waves) to evaluate the crack density. Although the equation for the crack density includes the ratio of the P–to–$S^{\perp}$ vertical velocities that cannot be obtained from P–wave data, the influence of the realistic errors in the velocity ratio is not that significant. The main application of this P–wave inversion algorithm is to identify pronounced anomalies of the crack density corresponding to "sweet spots" in fractured reservoirs. Another way to carry out the inversion for the crack density is to combine P– and $S^{\perp}$–wave NMO velocities from horizontal reflectors. The Thomsen's anisotropic coefficients and the crack density can be obtained from P and $S^{\perp}$ moveout in the single symmetry plane normal to the cracks if one of the vertical velocities is available.

While this work provides an analytic basis for moveout inversion in azimuthally anisotropic media, implementation of the algorithms outlined above may encounter some practical difficulties. The accuracy of the Dix differentiation, needed to obtain the NMO velocity in any individual layer, becomes inadequate for thin layers. For reservoirs with a relatively small thickness, the azimuthal variation of the reflection coefficient may be a more reliable diagnostic of azimuthal anisotropy. Also, the influence of vertical transverse isotropy (e.g., due to thin bedding) distorts the NMO equations derived for HTI media. Combination of vertical and horizontal transverse isotropy leads to a medium with orthorhombic symmetry that requires a special treatment not discussed here. If the azimuthal velocity variations in orthorhombic media are caused by vertical cracks, the difference in the NMO velocities in the two vertical symmetry planes can still be related to the crack density.

Wave propagation in the vertical symmetry planes of HTI media can be described using either isotropic equations (for the plane normal to the symmetry axis) or the formalism developed for vertical transverse isotropy (for the plane that contains the symmetry axis). In the latter plane, two different "equivalent" VTI models should be used, one responsible for $P - S^{\perp}$–waves and the other – for the wave $S^{\parallel}$. Time-related processing of P–waves in the plane containing the symmetry axis is governed by the zero-dip NMO velocity $V_{\text{nmo}}(0)$ and the parameter $\eta^{(V)}$ introduced by Alkhalifah and Tsvankin (1995) for vertical transverse isotropy. Although the values of the Thomsen parameters of the "equivalent" VTI medium are extremely uncommon for vertical transverse isotropy (e.g., $\epsilon^{(V)}$ is negative), time-related processing of P–wave data can still be performed by means of NMO, DMO, and migration algorithms developed for vertical transverse isotropy (e.g., Alkhalifah and Tsvankin, 1995). The anisotropic parameters recovered from moveout data make it possible to process data in off-symmetry planes as well. However, this processing cannot be performed without a proper treatment of the 3–D relation between phase and group velocities, which is not accounted for by VTI algorithms.

Although the moveout equation derived here is limited to horizontal transverse isotropy, the same approach can be used to study the azimuthal dependence of NMO velocities from both horizontal and dipping reflectors in more complicated azimuthally anisotropic media without performing ray tracing.

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APPENDIX A: AZIMUTHALLY-DEPENDENT NMO VELOCITY IN HTI MEDIA

Here, the approach suggested by Tsvankin (1995a) for moveout analysis in symmetry planes of anisotropic models is extended to an arbitary incidence plane in transversely isotropic media with a horizontal symmetry axis. Suppose the symmetry axis makes the angle α with the common-midpoint (CMP) line (Figure A-1). Normal-moveout (NMO) velocity is defined on CMP gathers as

$$V_{\text{nmo}}^2 = \lim_{x \to 0} \frac{d(x^2)}{d(t^2)}, \tag{A-1}$$

where x is the source-receiver offset and t is the two-way traveltime.

The derivation below is limited to the relatively simple case of horizontal reflectors, but the same approach can be used to find NMO velocity for reflections from dipping interfaces. Since a horizontal reflector represents a symmetry plane in HTI media, the group-velocity (ray) vector of any pure (non-converted) reflected wave is the mirror image of the incident ray with respect to the horizontal plane. This means that the incident and reflected *rays* (SO and OR in Figure A-1), are confined to the incidence (sagittal) plane, even if this plane is not a plane of symmetry. Furthermore, since the incident and reflected rays lie in the incidence plane and make the same angle with the reflector, they also make the same angle with vertical, and there is no reflection point dispersal on CMP gathers. However, the *phase-velocity* vectors of the incident and reflected waves may deviate from the incidence plane, while still being symmetric with respect to the reflector.

NMO velocity for a horizontally homogeneous medium above the reflector is convenient to evaluate using the equation given by Hale et al. (1992):

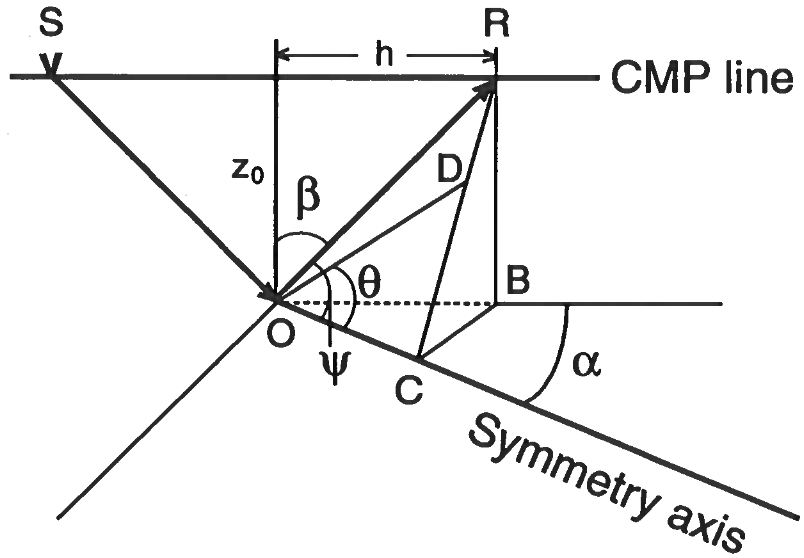

FIG. A-1. Geometry of the group- and phase-velocity vectors for reflected waves in HTI media. The incident (SO) and reflected (OR) group-velocity vectors (rays) are confined to the incidence plane. The phase-velocity vector (direction OD) corresponding to reflected ray OR lies in the plane formed by OR and the axis of symmetry. Triangle RCB defines the plane normal to the symmetry axis.

$$V_{\rm nmo}^2 = \frac{2}{t_0} \lim_{h \to 0} \left[\frac{d}{dh} \left(\frac{dt}{dh} \right) \right]^{-1} , \tag{A-2}$$

where $h = x/2$ is half the source-receiver offset, t_0 is the two-way traveltime along the zero-offset ray, and t is the one-way traveltime from the zero-offset reflection point to the receiver. In the case of a horizontal reflector beneath HTI media, both the phase- and group-velocity (ray) vectors of the zero-offset reflection are vertical. Note that the zero-offset ray is not necessarily vertical for other azimuthally anisotropic models, even for horizontal reflectors.

Equation (A-2) was derived under the assumption that the specular reflection point does not change with offset. As discussed above, this assumption is satisfied for our model; moreover, reflection point dispersal has no influence on NMO velocity because it contributes only to the quartic and higher-order terms of the traveltime series (Hubral and Krey, 1980, Appendix D; Tsvankin, 1995a).

Since the derivative dt/dh represents the apparent slowness on the CMP gather, it is equal to the projection of the slowness vector on the CMP line:

$$p_{\rm h} = \frac{dt}{dh} ,$$

and the NMO velocity [equation (A-2)] can be rewritten as

$$V_{\rm nmo}^2 = \frac{2}{t_0} \lim_{h \to 0} \frac{dh}{dp_{\rm h}}, \tag{A-3}$$

Equation (A-3) remains valid for the case when the rays, as well as the slowness vectors of the incident and reflected waves, diverge from the incidence plane. Thus, it can be applied to much more complicated problems than the one considered here.

Introducing the group angle β in the incidence plane (Figure A-1) and substituting $h = z_0 \tan\beta$ and $z_0 = V_{vert}\, t_0/2$ (V_{vert} is the vertical velocity) yields

$$V_{\rm nmo}^2 = V_{vert} \lim_{\beta \to 0} \frac{d\tan\beta}{dp_{\rm h}}. \tag{A-4}$$

It is convenient to represent β and $p_{\rm h}$ as functions of the phase angle θ with the symmetry axis (Figure A-1). Note that the phase-velocity vector in transversely isotropic media always lies in the plane formed by the symmetry axis and the group-velocity vector. Equation (A-4) then becomes

$$V_{\rm nmo}^2 = V_{vert} \lim_{\theta \to 90^\circ} \frac{d\tan\beta}{d\theta} \left(\frac{dp_{\rm h}}{d\theta}\right)^{-1}. \tag{A-5}$$

Next, it is necessary to estimate both derivatives in equation (A-5). From simple trigonometry (Figure A-1),

$$\sin\beta = \frac{\cos\psi}{\cos\alpha},$$

where ψ is the group angle of ray OR with the symmetry axis. Then

$$\tan\beta = \frac{1}{\tan\psi \sqrt{1 - \frac{\sin^2\alpha}{\sin^2\psi}}}. \tag{A-6}$$

The group angle ψ can be expressed through the phase angle θ and phase velocity $V(\theta)$ as (Thomsen, 1986)

$$\tan\psi = \frac{\tan\theta + \frac{1}{V}\frac{dV}{d\theta}}{1 - \frac{\tan\theta}{V}\frac{dV}{d\theta}}. \tag{A-7}$$

Differentiating $\tan\psi$ with respect to θ yields (Tsvankin, 1995a)

$$\frac{d\tan\psi}{d\theta} = \frac{1 + \frac{1}{V}\frac{d^2V}{d\theta^2}}{\cos^2\theta \left(1 - \frac{\tan\theta}{V}\frac{dV}{d\theta}\right)^2}. \tag{A-8}$$

Using equations (A-6) and (A-8), we obtain the first derivative in equation (A-5):

$$\frac{d\tan\beta}{d\theta}(\theta = \psi = 90^\circ) = -\frac{1}{\cos\alpha}\left(1 + \frac{1}{V}\frac{d^2V}{d\theta^2}\right), \tag{A-9}$$

with the anisotropic term $\frac{1}{V}\frac{d^2V}{d\theta^2}$ to be evaluated at $\theta = 90^\circ$.

Now we have to find the relation between the projection of the slowness vector on the CMP line (p_h) and the phase angle θ. The slowness vector (which is parallel to OD in Figure A-1) can be decomposed into two vectors parallel to sides OC and CD of triangle OCD. Taking into account that

$$\cos(< RCB) = \frac{\tan\beta\,\sin\alpha}{\sqrt{1 + \tan^2\beta\,\sin^2\alpha}} = \frac{\tan\alpha}{\tan\psi},$$

and projecting each of the two components on the CMP line, we get

$$p_\text{h} = \frac{1}{V}\left(\cos\theta\,\cos\alpha + \sin\theta\,\sin\alpha\,\tan\alpha/\tan\psi\right), \tag{A-10}$$

with $\tan\psi$ given by equation (A-7).

Evaluating the derivative of equation (A-10) with respect to θ yields

$$\frac{dp_\text{h}}{d\theta}(\theta = \psi = 90^\circ) = -\frac{1}{V\,\cos\alpha}\left(1 + \frac{\sin^2\alpha}{V}\frac{d^2V}{d\theta^2}\right). \tag{A-11}$$

Finally, we obtain NMO velocity by substituting equations (A-9) and (A-11) into equation (A-5):

$$V_\text{nmo}^2 = V_{vert}^2\,\frac{1 + \frac{1}{V}\frac{d^2V}{d\theta^2}}{1 + \sin^2\alpha\left[\frac{1}{V}\frac{d^2V}{d\theta^2}\right]}, \tag{A-12}$$

where both the phase velocity V and its second derivative should be evaluated at the phase angle $\theta = 90^\circ$.

APPENDIX B: NMO VELOCITY ALONG THE SYMMETRY DIRECTION

Here, the P–wave normal-moveout velocity on a CMP gather parallel to the symmetry axis is derived directly from the phase-velocity equation, without using the analogy between vertical and horizontal transverse isotropy discussed in the main text. If the symmetry direction is parallel to the CMP line ($\alpha = 0$), equation (A-12) reduces to

$$V_\text{nmo}^2 = V_{vert}^2\left(1 + \frac{1}{V}\frac{d^2V}{d\theta^2}\right). \tag{B-1}$$

Since the term $\frac{1}{V}\frac{d^2V}{d\theta^2}$ should be calculated at vertical incidence ($\theta = 90°$), equation (B-1) coincides with NMO formula (57) of Tsvankin (1995a) adapted for horizontal reflectors. To evaluate NMO velocity (B-1) for the P–wave, we use the exact expression for P–wave phase velocity in Thomsen notation given by Tsvankin (1995b):

$$\begin{aligned}\frac{V^2(\theta)}{V_{P0}^2} &= 1 + \epsilon \sin^2\theta - \frac{f}{2} \\ &+ \frac{f}{2}\sqrt{\left(1 + \frac{2\epsilon \sin^2\theta}{f}\right)^2 - \frac{8(\epsilon - \delta)\sin^2\theta\cos^2\theta}{f}}\,. \end{aligned} \tag{B-2}$$

where

$$f \equiv 1 - V_{S0}^2/V_{P0}^2\,,$$

V_{P0} and V_{S0} are the the P– and S–wave velocities respectively in the symmetry (horizontal) direction.

Differentiating $V_P(\theta)$ from equation (B-2) twice with respect to θ, we find

$$\frac{d^2V}{d\theta^2}(\theta = 90°) = -\frac{2V_{P0}}{\sqrt{1+2\epsilon}}\left(\epsilon + \frac{\epsilon - \delta}{1 + \frac{2\epsilon}{f}}\right)\,. \tag{B-3}$$

The P–wave vertical velocity for horizontal transverse isotropy is given by

$$V_{Pvert} = V_{P0}\sqrt{1+2\epsilon}\,. \tag{B-4}$$

Substitution of equations (B-4) and (B-3) into NMO expression (B-1) yields

$$V_{\mathrm{nmo}}(P - \mathrm{wave}) = V_{P0}\sqrt{1 - \frac{2(\epsilon - \delta)}{1 + \frac{2\epsilon}{f}}}\,. \tag{B-5}$$